Abstracts

Secular dynamics of S-type orbits in binary star systems

Eduardo Andrade-Ines

Binary stars are frequent in the universe, composing approximately 50% of main sequence stars (Abt 1979; Duquennoy & Mayor 1991; Raghavan et al. 2010). Due to inherent difficulties in monitoring the radial velocities of multi-star systems, these have not been primary targets in exoplanet surveys (Eggenberger & Udry 2010). Still, at least 10% of the currently known extra-solar planets are hosted in binary stars (Roell et al. 2012). Secular perturbations rule the dynamics of subjects such as the formation (Heppenheimer 1978, Giuppone et al. 2011), the long term evolution and habitability (Andrade-Ines & Michtchenko 2014) of planets in binary star systems. However, the large mass of the disturbing body and the frequent high values of the eccentricities in such systems limits the use of the classical secular theories based on the Laplace expansion of the disturbing function (e.g. Brouwer & Clemence 1961). In this talk, I will present the development of a general approach for the coplanar second-order secular model based on Lie-series canonical perturbation theory. Lastly, I will compare the results of the developed models to numerical integrations of the N-body problem and determine the limits of applicability of the different secular models for planetary orbits in S-type binary systems.

KAM theory for secondary tori in mechanical systems

Luca Biasco

We prove the persistence of secondary tori for general mechanical systems. Work in collaboration with L. Chierchia.

KAM theory and quasi-periodic attractors of some dissipative systems

Alessandra Celletti

The talk concerns recent results about the stability of some dissipative systems, precisely conformally symplectic systems. We describe a suitable KAM theory, which allows to prove the persistence of invariant attractors. The proof is constructive and it provides efficient algorithms to evaluate the breakdown threshold of quasi-periodic attractors. Applications to model problems, with interest also in Celestial Mechanics, are provided. The talk refers to works done in collaboration with R. Calleja and R. de la Llave.

Construction of q-tori in the FPU model with Poincaré-Lindstedt series

Helen Christodoulidi

In this talk we will present a construction of low-dimensional invariant tori in the phase space of the Fermi-Pasta-Ulam model using Poincaré-Lindstedt series. We call these solutions q-tori, since they possess exponentially localized profiles in q-space and share similar features to periodic solutions known as q-breathers. We probe numerically the convergence properties as well as the level of precision of our computed series. We develop an additional algorithm in order to systematically identify values of the incommensurable frequencies used as an input in the Poincaré-Lindstedt series construction of q-tori corresponding to progressively higher values of the energy.

Special Lecture:
The history of the Third Integral (a personal account)

George Contopoulos

We consider various developments in the history of the third integral of motion in a galaxy. In particular, we mention the discovery of particular forms of the third integral in resonant cases, the generation of chaos due to resonance overlap, the nonlinear theory of spiral density waves, applications to relativity and cosmology and the new problems of 3 degrees of freedom. Then we emphasize the recent developments concerning order and chaos in quantum mechanics and the use of the third integral in finding chaotic orbits.

Bifurcations and geometry in 1:1:-2 resonant Hamiltonian systems

Kostantinos Efstathiou

We consider integrable Hamiltonian systems in three degrees of freedom near an equilibrium in 1:1:-2 resonance. The integrability is due to the existence of two symmetries: a symmetry coming from the periodic motion of the quadratic part and an imposed rotational symmetry about the vertical axis. We find a rich bifurcation diagram containing three parabolas of Hamiltonian Hopf bifurcations that join at the origin and we describe in detail the monodromy of the resulting ramified 3-torus bundle.

Computational Hamiltonian perturbation theory: prospects and challenges

Christos Efthymiopoulos, Ugo Locatelli, Giuseppe Pucacco

The advent of new powerful computational techniques and resources creates new horizons in the development of canonical perturbation theory. From computer-assisted proofs to specific applications in realistic systems of physical interest, a main challenge for perturbation theory is to identify those perturbative schemes and techniques whose implementation becomes most efficient in the computer. This last question is not irrelevant to the development of the theory itself. Issues like the accumulation of small divisors, the effective splitting of the Hamiltonian in orders of smallness depending on one or more small parameters, norm estimates and iterative lemmas, and, eventually, the speed of convergence or the optimal asymptotic behavior of normal form series, crucially affect the way that the actual computations are organized in the computer. We will discuss some examples of the interplay between advances on the theoretical and computational sectors of canonical perturbation theory, as well as how this interplay shapes the prospects of this whole subject. We will thus summarize a central motivation for our decision to organize the present meeting.

Periodic motion for a solar sail close to an asteroid

Ariadna Farrés-Basiana

The Augmented Hill problem is a 3DoF Hamiltonian system used to understand the motion of a solar sail in the vicinity of an asteroid. This model includes the gravitational effects of both the Sun and the asteroid, as well as the solar radiation pressure due to the solar sail. This last one depends on 4 parameters: two dimensionless parameters related to the sail performance β, ρ, and two angles defining the sail orientation α, δ.

The classical Hill problem (no solar sail) has been widely studied in the past, and its non-linear dynamics is well know. The system has two equilibrium (L_1 and L_2) and several families of periodic and quasi-periodic orbits around them. In this talk we will discuss how the extra effect of the solar sail affects the equilibrium points and periodic orbits that appear in the system. We will describe the richness of the dynamics when we vary the different parameters in the system.

This study is not only motivated for the richness in the system, but also it is well suited for practical mission applications. Where having a good understanding on the dependence of the phase space portrait with respect to the sail parameters will help enabling practical mission applications.

A study of the lunisolar secular resonances for space debris by using the Hamiltonian formalism

Cătălin Galeş

Understanding the dynamics and evolution of small bodies moving in the space surrounding the Earth is a subject of large interest, due to the awareness of the problems that space debris can cause in the future. Therefore, a detailed investigation of the effects induced by each perturbing force, including the gravitational influence of the Sun and Moon, is nowadays mandatory. A very important role in the long-term dynamics of space debris is played by resonances. Two types of resonance affect the motion of space debris: tesseral resonances, occurring when there is a commensurability between the Earth's rotation period and the orbital period of the space debris, and lunisolar resonances, which involve commensurabilities among the slow frequencies of orbital precession of the debris and the perturbing body. Tesseral resonances provoke variations of the semi–major axis on a time scale of the order of hundreds of days, while lunisolar resonances influence the evolution of the eccentricity and inclination on a much longer time scale, of the order of tens (or hundreds) of years. The purpose of this talk is to present some results related to lunisolar resonances. By addressing the topic using the Hamiltonian formalism, and by using tools and techniques of nonlinear dynamics, we discuss several dynamical phenomena at which a small object is being subject: overlapping of resonances, bifurcations, variation of the amplitude of resonances, chaotic variations of the orbital elements. Joint work with Alessandra Celletti and Giuseppe Pucacco.

The game of small divisors in normal forms

Antonio Giorgilli

My aim is to revisit the problem of small denominators in perturbation series. As a simple model, first investigated by Poincaré, I will consider the problem of linearization of a vector field in the neighbourhood of an equilibrium. Using an appropriate algorithm it is seen that the accumulation of small divisors is subjected to very strict rules. As a result, an improved version of the convergence estimates first found by Siegel is worked out. The same scheme applies also to the more challenging proof of Kolmogorov's theorem on invariant tori.

On the averaged restricted N-body problem with orbit-crossing singularities

Giovanni F. Gronchi

We consider the long term evolution of a small celestial body in the framework of the averaged restricted N-body problem. These bodies may represent the Sun, the solar system planets and an asteroid: only the motion of the asteroid is unknown. We assume that the osculating ellipse, representing the Keplerian trajectory of the asteroid, can cross the osculating trajectory of some planet during the evolution. If this happens we speak of an orbit-crossing. We shall review some recent results where the secular evolution of the orbital elements of the asteroid is computed by averaging the Hamiltonian equations of motion over the mean anomalies of the asteroid and the planets. The main difficulty is due to the orbit crossings with the planets: when such crossings occur, the averaged equations become singular. However, it is possible to define piecewise differentiable solutions by extending the averaged vector field beyond the singularity from both sides of the orbit crossing set. The averaging method can be used if no mean motion resonance with a planet occur. We shall also show how to deal with this problem in case of such resonances by using tools from perturbation theory: in particular we construct resonant normal forms with orbit-crossing singularities. Finally, we shall describe the behavior of the secular evolution of the distance between the Keplerian trajectories of the small body and the planet whose trajectory is crossed.

A guided visualization of the tube manifolds of the three body problem

Massimiliano Guzzo

The stable and unstable manifolds of the center manifolds of the Lagrangian equilibrium points L1, L2 play a key role in the understanding of the complicated dynamics of the circular restricted three-body problem. Using a technique based on the Fast Lyapunov Indicators, we have computed sample three-dimensional representations of the manifolds which show an original vista about their complicated development in the phase-space.

Hamiltonian oscillators in 1:±2:±4 resonance

Heinz Hanßmann

The 1:2:4 resonance is one of the four definite resonances of genuinely first order and thus known to be non-integrable. The frequency ratios provide unfolding parameters (but note that the dynamic phenomena can also occur in a single system of 6 or more degrees of freedom). The indefinite versions of the resonance do not require the equilibrium to be a local extremum of the Hamiltonian.

Normalization yields a normal form approximation and the resulting (non-integrable) system can be reduced to 2 degrees of freedom. The non-trivial isotropies of the two coupled 1: ± 2 resonances prevent the reduced phase space from being a smooth manifold but the dynamics on the singular part is in fact easier to understand. On the regular part of the reduced phase space the distribution of equilibria turns out to be determined by a single polynomial of degree 4. These are the relative equilibria that determine the behaviour of the 3 normal modes when passing through the resonance.

On the (computer-assisted) application of an a posteriori KAM theorem

Alex Haro

We present a methodology to rigorously validate a given approximation of a quasi-periodic Lagrangian torus of an exact symplectic map. The approach consists in verifying the hypotheses of an a-posteriori KAM theorem based of the parameterization method (following Rafael de la Llave and collaborators). A crucial point of our imprementation is an Approximation Lemma that allows us to control the norm of real-analytic periodic functions using their discrete Fourier transform. An outstanding consequence of this approach is that the computational cost of the validation is assymptotically equivalent of the cost of the numerical computation of invariant tori using the parameterization method. We illustrate the methodology with several examples, as the standard map, and the Froeschlé map. This is a joint work with Jordi-Lluís Figueras and Alejandro Luque.

The 1:1 resonance of the augmented Hill problem

Angel Jorba

The Augmented Hill problem is a Hamiltonian system with three degrees of freedom used to model the dynamics of a Solar sail in the neighbourhood of an asteroid. The model can be described as the classical Hill model plus an extra term to account for the Solar radiation pressure. It is remarkable that, for some values of the parameters, the linear frequencies of one of the equilibrium points are in a 1:1 resonance. In the talk we will give a numerical description of the dynamics in the neighbourhood of this resonance. This is a joint work with A. Farrés.

Perturbation theory at arbitrary expansion points - Applications in Celestial Mechanics

Christoph Lhotka

Classical expansion techniques rely on sophisticated algorithms to cope with the enormous amount of terms in the series developments. In this talk we demonstrate the usefulness and limitations of generic expansion points in Celestial Mechanics with applications to artificial satellite theory, dust and small body dynamics.

Extension of the Laplace-Lagrange secular theory to extrasolar systems

Anne-Sophie Libert

In contrast to the quasi-circular planetary orbits of the Solar System, the exoplanets usually describe true ellipses with high eccentricities. The classical Laplace-Lagrange perturbation theory uses the circular approximation as a reference, and can thus be doubtful for these systems. We aim to show here that perturbation theory reveals very efficient for describing the long-term evolution of the extrasolar systems. More precisely, we study the long-term evolution of two-planet extrasolar systems by extending the Laplace-Lagrange theory for three categories of systems: (i) secular systems, whose long-term evolution is accurately described by an extension of the classical Laplace-Lagrange theory to a high order in eccentricities; (ii) systems that are near a mean-motion resonance, for which an extension of the Laplace-Lagrange secular theory to order two in the masses is required; (iii) systems that are really close to or in a mean-motion resonance, where a resonant model has to be used. In the first two cases, we determine the fundamental frequencies of the motion and compute precisely the long-term evolution of the Keplerian elements with a totally analytical method, based on Lie transforms. For the last category of systems, we discuss how the long-term evolution can accurately be reproduced by including appropriate resonant combinations of the fast angles into the Laplace-Lagrange expansion. Joint work with M. Sansottera.

Exploring the feedback of asymmetric jets on the orbital motions in protostellar discs

Ugo Locatelli

The eccentricities of the orbits in extrasolar multi-planetary systems can be much larger than those in our solar system. This is somewhat surprising, as large eccentricities can prevent the overall stability. Motivated by this evidence we have started investigating if such peculiar orbital properties can be a legacy of the primordial disk conditions. In particular, we explore the influence on the parent disk of asymmetric jets, as they are observed to be in a large fraction of cases. Spectral diagnostics reveals that in these jets the two lobes generally have different linear momentum fluxes. This produces a dynamical feedback on the inner part of the disk, from where the jet is accelerated. Here we describe the first steps of a new research project aimed at the investigation of such scenario, using a novel approach based on the methods of celestial mechanics. We illustrate the first results of numerical simulations designed for a simple model in which an asymmetric disk-wind acts on annuli of material orbiting in the accretion disk, and we discuss the implications for the disk structure. Joint work with F. Bacciotti, R.I. Páez, L. Podio and M. Volpi.

Symmetric resonances in Hamiltonian systems

Antonella Marchesiello

Symmetric resonances play a prominent role in nonlinear Hamiltonian dynamics, as they appear in a great variety of applications, from molecular dynamics to astrodynamics. With an eye to applications to galactic dynamics, we focus on systems with unperturbed frequencies close to a resonant ratio and that are invariant under reflection symmetries with respect to the coordinate axes. By combining singularity theory with geometric methods, we investigate the rich structure of these systems and, by varying their physical parameters, we describe their bifurcation sequences around some of the most relevant resonances. Joint work with G. Pucacco.

Quasi periodic coorbital motions

Laurent Niederman

The motions of the satellites Janus and Epimetheus around Saturn are among the most intriguing in the solar system. These satellites exchange their orbits every four years. We give a rigorous proof (and up to our knowledge, the first one) of the existence of stable orbits of this kind in the three body problem thanks to KAM theory.

Reduction of resonant Hamiltonians

Jesús Palacián

We deal with the analysis of Hamiltonian systems from a qualitative point of view, establishing the existence, stability and bifurcations of periodic solutions as well as the existence of some invariant tori. Our purpose is to illustrate the use of singular reduction on this problem. Reduction lowers the dimension of the problem under study; so, given that our first test problem is a two-degree-of-freedom Hamiltonian system in $\mathbf{R}^4$, it will be reduced to a Hamiltonian system of one degree of freedom on a two-dimensional surface called orbifold. The restricted three-body problem is considered as a benchmark for many years, and in particular there has been a bunch of works to obtain periodic solutions and related invariant sets around the equilibrium points L_4 and L_5. Then we shall handle the n degrees of freedom case where the polynomial invariants needed in the reduction theory are determined using an algorithm based on integer programming to obtain a Hilbert basis for a given resonance. The cardinality of the Hilbert basis is not known a priori but it is lower-bounded by n^2. After computing the Hilbert basis we use Gröbner bases and the division algorithm for multivariate polynomials to deal with the equations of motion in terms of the invariants. Besides we build the orbifold from the invariants and the constraints among them. Our aim is to reconstruct the periodic solutions of the full system as well as the n-dimensional KAM tori from the critical points in the orbifold. To achieve it we use local symplectic coordinates around these points. We apply the theory getting some periodic solutions in resonant Hamiltonian systems of n degrees of freedom. This is a joint work with Ken R. Meyer and Patricia Yanguas.

Hamiltonian perturbation techniques at the thermodynamic limit: adiabatic invariants and normal forms for the Klein-Gordon chain

Simone Paleari

An important challenge in Hamiltonian dynamics is the development of a perturbation theory for Hamiltonian systems with an arbitrarily large number of degrees of freedom, and in particular in the thermodynamic limit. Indeed, motivated by the problems arising in the foundations of Statistical Mechanics, it is relevant to consider large systems with non vanishing energy per particle (which corresponds to a non zero temperature in the physical model).

Being interested in the low temperature regime, it is foreseeable the use of perturbation theory to exploit the presence of a small parameter like the specific energy. Unfortunately, it is a well known limit of the classical results of this theory (like KAM or Nekhoroshev theorem) to suffer a bad dependence on the number of degrees of freedom, often resulting in void or non applicable statements in the thermodynamic limit.

I will try to present (one or both of) a couple of results in the direction of the aforementioned goals. The model considered is in both cases a finite but arbitrarily large Klein-Gordon chain, with periodic boundary conditions. Such results can be described as follows:

- We construct an extensive adiabatic invariant in the thermodynamic limit. In particular, given a fixed and sufficiently small value of the coupling constant a, the evolution of the adiabatic invariant is controlled up to times scaling as $\beta^{1/a}$ for any large enough value of the inverse temperature β. The time scale becomes a stretched exponential if the coupling constant is allowed to vanish jointly with the specific energy. The adiabatic invariance is exhibited by showing that the variance along the dynamics, i.e. calculated with respect to time averages, is much smaller than the corresponding variance over the whole phase space, i.e. calculated with the Gibbs measure, for a set of initial data of large measure. All the perturbation constructions and the subsequent estimates are consistent with the extensive nature of the system.

- In the limit of small couplings in the nearest neighbor interaction, and small (total or specific) energy, a high order resonant normal form is constructed with estimates uniform in the number of degrees of freedom. In particular, the first order normal form is a generalized discrete nonlinear Schrödinger model, characterized by all-neighbors coupling with exponentially decaying strength.

Joint work with Antonio Giorgilli and Tiziano Penati. References:
Annales Henri Poincaré, **16**, 897–959 (2015)
Annali di Matematica Pura ed Applicata, **195**, 133–165 (2016).

Secular dynamics of multi-planetary systems around binary stars

Philippe Robutel

After having presented a canonical coordinate system adapted to the study of the motion of n-1 planets orbiting a binary star. I will show how to expand the secular part of this Hamiltonian in terms of (canonical) elliptic elements. The last part of the talk will be dedicated to the secular dynamics of the planar circumbinary systems. After having recalled the main dynamical features of the secular circumbinary three-body problem, a specific secular resonance and its associated bifurcation will be studied in the case of two planets orbiting a binary star.

High-order control for symplectic maps

Marco Sansottera

We revisit the problem of introducing an a priori control for devices that can be modeled via a symplectic map in a neighborhood of an elliptic equilibrium. Using a technique based on Lie transform methods we produce a normal form algorithm that avoids the usual step of interpolating the map with a flow. The formal algorithm is completed with quantitative estimates that bring into evidence the asymptotic character of the normal form transformation. Then we perform an heuristic analysis of the dynamical behavior of the map using the invariant function for the normalized map. Finally, we discuss how control terms of different orders may be introduced so as to increase the size of the stable domain of the map. The numerical examples are worked out on a two dimensional map of Hénon type.

Dynamics of the Didymos binary asteroid - target of the AIDA mission

Kleomenis Tsiganis

In this talk I will describe the proposed mission AIDA, which is a joint mission by ESA (AIM) and NASA (DART), currently under Phase A/B1 study, and the work done by the Working Group on dynamical and physical properties of the system. The purpose of the mission is to rendez-vous with the binary NEA Didymos and impact the small ($\sim$ 150m) satellite, in order to induce a measurable (even from ground) deflection of its orbit around the primary ($\sim$ 800 m). The AIM spacecraft will provide useful information during a reconnaissance phase prior to impact, while DART will impact "Didy-moon" at $\sim$ 7 km/s, inducing a change in orbital period of $\sim$ 200 sec. Precise prior knowledge of the complicated dynamics of the system as well as predictions on the outcome of the impact are necessary for mission design. I will describe the numerical model that we use to study the dynamics as well as the classification of solutions (orbital and rotational motion) that we are getting, as functions of the (unknown) Didy-moon's shape. Of particular interest are solutions that lead to strong excitation of free libration, for sufficiently large asphericities of the moon. I will also show how analytical models can capture the essential features of these solutions and discuss what needs to be done in terms of perturbation theory, in order to achieve an accurate analytical approximation of the dynamics.

Quantum transition state theory

Holger Waalkens

A system displays reaction type dynamics if its phase space possesses bottleneck type structures. Such a system spends a long time in one phase space region (the region of 'reactants'), and occasionally finds its way through a bottleneck to another phase space region (the region of 'products'), or vice versa. In Hamiltonian systems such bottlenecks are induced by equilibrium points of saddle-center-…-center type ('saddles' for short). The most widely used method to compute reaction rates is Transition State Theory which has its origin of conception in chemistry where it was invented by Wigner, Eyring and Polanyi in the

1930's. The main idea here is to compute the reaction rate from the flux through a dividing surface placed in the bottleneck (or in chemical terms 'transition state') region. In order not to overestimate the rate the dividing surface needs to have the so-called 'no-recrossing' property which means that it is crossed exactly once by reactive trajectories and not crossed at all by nonreactive trajectories. The construction of such a dividing surface has posed a major problem in Transition State Theory since its invention in the 1930's. In the first part of my talk I will discuss in detail the phase space structures which govern the dynamics 'across' saddles, and how they can be computed from a normal form. This implies the construction of a dividing surface without recrossing which, as we will see, is 'spanned' by a normally hyperbolic invariant manifold. In the second part of the talk I will discuss the implications of the classical phase space structures for the quantum mechanics of reactions. We will see that a quantization of the classical normal form leads to an efficient algorithm to compute quantum reaction rates and the associated quantum resonances. The talk summarizes joint work with Roman Schubert and Stephen Wiggins from Bristol University.

Invariant tori in the spatial three-body problem

Patricia Yanguas

We deal with the spatial three-body problem in the various regimes where the Hamiltonian is split as the sum of two Keplerian systems plus a small perturbation. By averaging over the mean anomalies, truncating higher-order terms and using singular reduction theory we get a one-degree-of-freedom Hamiltonian system. Departing from the analysis concerning the relative equilibria of this reduced system, we carry out the reconstruction of the KAM tori surrounding the motions associated to each elliptic equilibrium. The existence of five-dimensional KAM tori for the spatial three-body problem is established. These tori surround various types of motions, from circular to near rectilinear, passing through coplanar or perpendicular. This is a joint work with Jesús F. Palacián and Flora Sayas.
