From inflation to dark energy through a dynamical Λ: an attempt at alleviating fundamental cosmic puzzles[1]

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(Submission date: March 29, 2013)

Abstract

After decades of successful hot big-bang paradigm, Cosmology still lacks a framework in which the early inflationary phase of the universe smoothly matches the radiation epoch and evolves to the present 'quasi' de Sitter spacetime. No less intriguing is that the current value of the effective vacuum energy density is vastly smaller than the value that triggered inflation. In this *Essay* we propose a new class of cosmologies capable of overcoming, or highly alleviating, some of these acute cosmic puzzles. Powered by a decaying vacuum energy density, the spacetime emerges from a pure nonsingular de Sitter vacuum stage, "gracefully" exits from inflation to a radiation phase followed by dark matter and vacuum regimes, and, finally, evolves to a late time de Sitter phase.

[1] Essay written for the Gravity Research Foundation 2013 Awards Essays on Gravitation

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In the standard view of cosmology, matter and space-time emerged from a singularity and evolved through four different regimes: inflation, radiation, dark matter and dark energy dominated eras. In the radiation and dark matter dominated stages, the expansion of the Universe decelerates while the inflation and dark energy eras are accelerating phases. So far there is no clear cut connection between these accelerating periods. More intriguing, the substance (if any) driving the present accelerating stage remains a complete mystery. It has been called Dark Energy (DE), and its formulation embodies a large number of models trying to describe its nature and dynamical properties.

The top ten DE candidate is the cosmological constant, Λ, or vacuum energy density ($\rho_\Lambda \equiv \Lambda/8\pi G$), several times resurrected since Einstein introduced it 96 years ago. Although its concept is plagued with the cosmological constant [1] and coincidence problems [2], most alternative DE models present similar difficulties, and none of them can scape from extreme fine tuning and/or insufficient fundamental motivation [3].

It is remarkable that the Einstein field equations (plus the Cosmological Principle) do not prevent Λ to evolve with cosmic time or a function of it. While its precise functional form has not yet been determined, quantum field theory (QFT) in curved spacetime singles out the general form of the evolution of the vacuum energy density, ρ_Λ, as a function of the Hubble rate. Specifically, it suggests a renormalization group (RG) equation in which the rate of change of ρ_Λ with $H(t)$ contains only even powers of H (because of the covariance of the effective action) [4–7]:

$$\frac{d\rho_\Lambda}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_i \left[a_i M_i^2\, H^2 + \, b_i\, H^4 + c_i \frac{H^6}{M_i^2} \, + ... \right] , \tag{1}$$

where the (dimensionless) coefficients receive loop contributions from boson and fermion (hereafter b and f) matter fields of different masses M_i. Obviously, the expansion (1) converges very fast at low energies, where H is rather small – certainly much smaller than any particle mass. No other term beyond H^2 (not even H^4) can contribute significantly on the *r.h.s.* of equation (1) at any stage of the cosmological history below the GUT scale M_{GUT}, typically a few order of magnitude below the Planck scale $M_P \sim 10^{19}$ GeV. But in the very early universe (when H is also close, but below, the masses of the heavy fields $M_i \sim M_{GUT}$) the H^4 effects can also be significant, whereas the terms H^6/M_i^2 and above

are less and less important. Integrating the above equation we arrive at

$$\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha \, \frac{H^{n+2}}{H_I^n} \, , \tag{2}$$

where H_I is the Hubble parameter at the inflation. Here $n = 2$, but we leave it generical since the main results turn out to be independent of n, as we shall see. If we banish odd powers of H for the aforementioned reasons (see however [8]) the dominant part of the series (1) is expected to be naturally truncated at the H^4 term [9]. The coefficients $\nu = \frac{1}{6\pi} \sum_{i=f,b} c_i \frac{M_i^2}{M_P^2}$ and $\alpha = \frac{1}{12\pi} \frac{H_I^2}{M_P^2} \sum_{i=f,b} b_i$ receive contributions from all the matter particles and play the role of one-loop β-functions for the RG running. Both coefficients are predicted to be naturally small since $M_i^2 \ll M_P^2$ for all the particles, even for the heavy fields of a typical GUT below the Planck scale. In the case of ν an estimate within a generic GUT is found in the range $|\nu| = 10^{-6} - 10^{-3}$ [7]. Remarkably, this coefficient can also be observationally accessed. From a joint likelihood analysis of the recent supernovae type Ia data, the CMB shift parameter, and the BAO, one finds that the best fit value for ν for a flat universe is $|\nu| \lesssim \mathcal{O}(10^{-3})$ [10, 11], which is nicely in accordance with the theoretical expectations.

The equations of state for the dynamical vacuum and matter fluids are still $p_\Lambda(t) = -\rho_\Lambda(t)$ and $p = \omega\rho$ (ω constant), respectively. The overall conservation law in the presence of a dynamical Λ-term reads $\dot{\rho} + 3(1+\omega)H\rho = -\dot{\rho_\Lambda}$, entailing energy exchange between matter and vacuum. Combining this conservation equation with (2) and Friedmann's equation in flat space, $8\pi G(\rho + \rho_\Lambda) = 3H^2$, we obtain the differential evolution law for $H(t)$:

$$\dot{H} + \frac{3}{2}(1+\omega)H^2 \left[1 - \nu - \frac{c_0}{3H^2} - \alpha \left(\frac{H}{H_I}\right)^n\right] = 0 \, . \tag{3}$$

Interestingly enough it admits the constant solution $H = H_I[(1-\nu)/\alpha]^{1/n}$, corresponding to an inflationary regime in the very early universe, i.e. when $H^2 \gg c_0$. On the other hand, at late times, when $H \ll H_I$, and for $\nu \ll 1$ we have that $\Lambda \approx c_0$, which behaves as an effective cosmological constant. In a nutshell, the phases of the decaying vacuum cosmology (2) are the following: **(i)** the universe starts from an unstable inflationary phase powered by the huge value H_I (presumably connected to a GUT scale near M_P), **(ii)** it next enters a deflationary period (with a massive production of relativistic particles) triggering the radiation epoch, followed by the conventional cold matter epoch, and, finally, **(iii)** the vacuum energy density effectively appears today as a slowly varying dynamical DE, thanks

to the $3\nu H^2$ term (with $|\nu| \ll 1$) in Eq. (2), which mildly corrects the phenomenology of the standard ΛCDM model.

Let us first discuss the transition from an initial de Sitter stage to the standard radiation phase ($\omega = 1/3$). The Hubble function of this model in the early universe (when c_0 can be neglected in front of H^2) follows from direct integration of Eq.(3):

$$H(a) = \left(\frac{1-\nu}{\alpha}\right)^{1/n} \frac{H_I}{\left[1 + D\, a^{2\,n\,(1-\nu)}\right]^{1/n}}\,. \tag{4}$$

Here $D = a_\star^{-2\,n\,(1-\nu)} \left[\frac{1-\nu}{\alpha}\left(\frac{H_I}{H_\star}\right)^n - 1\right]$ is fixed from the condition $H(a_\star) \equiv H_\star$, where $a_\star$ is the scale factor at the transition time ($t_\star$) when the inflationary period ceases. Of special physical significance are the corresponding vacuum and radiation energy densities:

$$\rho_\Lambda(a) = \tilde{\rho}_I\, \frac{1 + \nu\, D\, a^{2n(1-\nu)}}{\left[1 + D\, a^{2n(1-\nu)}\right]^{1+2/n}}\,, \qquad \rho_r(a) = \tilde{\rho}_I\, \frac{(1-\nu) D\, a^{2n(1-\nu)}}{\left[1 + D\, a^{2n(1-\nu)}\right]^{1+2/n}}\,, \tag{5}$$

where $\tilde{\rho}_I \equiv [(1-\nu)/\alpha]^{2/n}\, \rho_I$, with $\rho_I = 3H_I^2/8\pi G$. For $D\, a^{2n(1-\nu)} \ll 1$ (the very early universe) Eq. (4) boils down to the aforesaid constant value solution $H \approx H_I[(1-\nu)/\alpha]^{1/n}$. In this period the vacuum energy density remains almost constant $\rho_\Lambda \approx \tilde{\rho}_I$, and the universe grows exponentially fast: $a(t) \propto \mathrm{e}^{\left(\frac{1-\nu}{\alpha}\right)^{1/n} H_I t}$ (the primeval de Sitter era). Right next the standard radiation dominated era emerges (see the inner panel of Fig. 1). Thermodynamically, since the model starts as a de Sitter spacetime, the most natural choice for the temperature is the Gibbons-Hawking temperature [12] of its event horizon. Thus the expansion proceeds isothermally at $T_I = H_I/2\pi$ during the initial de Sitter phase, with H_I of the order of the GUT scale M_{GUT} near the Planck mass (see [9] for $n = 2$).

The outcome of the above considerations is that for $D \neq 0$ the universe starts without a singularity. Furthermore, a light pulse beginning at $t = -\infty$ will have traveled by the cosmic time t a physical distance $d_H(t) = a(t) \int_{-\infty}^{t} \frac{dt'}{a(t')}$, which diverges, thereby implying the absence of particle horizons. As a result the local interactions may causally homogenize the whole universe.

Remarkably, for $D\, a^{2n(1-\nu)} \gg 1$ the solution (4) displays the behavior $H \sim a^{-2(1-\nu)}$ and so $a(t) \sim t^{1/2(1-\nu)}$. As $|\nu| \ll 1$, it is obvious that $a(t) \sim t^{1/2}$, signaling the onset of the standard radiation epoch. This is further confirmed upon inspecting the radiation energy density in (5), which decays as $\rho_r \sim a^{-4(1-\nu)} \sim a^{-4}$. Worth noticing is that the vacuum energy density follows a similar decay law $\rho_\Lambda \sim \nu a^{-4(1-\nu)}$, but it is suppressed by $\rho_\Lambda/\rho_r \propto \nu$ (with $|\nu| \ll 1$)

as compared to the radiation density. This insures that primordial nucleosynthesis will not be harmed at all. In short, a conventional radiation epoch is granted and a clue to the "graceful exit" from the inflationary stage seems feasible.

Although we cannot provide at this point the effective action for the model, we can at least mimic it through a scalar field (ϕ) model for the interacting DE [13, 14]. This can be useful for the usual phenomenological descriptions of the DE, and can be obtained from the correspondences $\rho_{\rm tot} \to \rho_\phi = \dot{\phi}^2/2 + V(\phi)$ and $p_{\rm tot} \to p_\phi = \dot{\phi}^2/2 - V(\phi)$ in Friedmann equations, with the result

$$V(a) = \frac{3H^2}{8\pi G}\left(1 + \frac{\dot{H}}{3H^2}\right) = \frac{\rho_I}{\alpha^{2/n}}\,\frac{1 + Da^{2n}/3}{(1 + Da^{2n})^{(n+2)/n}}\,, \tag{6}$$

where we have now neglected the small $\mathcal{O}(\nu)$ corrections. It is apparent that $V \sim \rho_I$ for $a \ll D^{-1/2n}$ (i.e. before the transition from inflation to deflation). However, when the transition is left well behind ($a \gg D^{-1/2n}$) the effective potential decreases in the precise form $V(a) \sim a^{-4}$, valid for all n, as it should in order to describe a radiation dominated universe – independently of the power n. This result corroborates, in the scalar field language, the correct transition to the radiation dominated epoch.

Finally, we briefly analyze the expanding universe at times after recombination, therefore consisting of dust ($\omega = 0$) plus the running vacuum fluid described by Eq.(2) with $H \ll H_I$. In this case the H^{n+2} term ($n > 1$) is completely negligible compared to H^2 and we have $\Lambda(H) = \Lambda_0 + 3\,\nu\,(H^2 - H_0^2)$, with $\Lambda_0 \equiv c_0 + 3\nu\,H_0^2$. Obviously, c_0 plays an essential role to determine the value of Λ, and since $|\nu| = \mathcal{O}(10^{-3})$ the H^2 dependence gives some remnant dynamics even today. Trading the cosmic time for the scale factor and using the redshift variable $1 + z = 1/a$ with the boundary condition $H(z = 0) = H_0$, one finds the solution of Eq.(3) for the late stages:

$$H^2(z) = \frac{H_0^2}{1-\nu}\left[(1 - \Omega_\Lambda^0)(1+z)^{3(1-\nu)} + \Omega_\Lambda^0 - \nu\right]\,, \tag{7}$$

where $\Omega_\Lambda^0 = \Lambda_0/3H_0^2 = 8\pi\,G\,\rho_\Lambda^0/3H_0^2$. The corresponding matter and vacuum energy densities read: $\rho_m(z) = \rho_m^0(1+z)^{3(1-\nu)}$ and $\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right]$. They deviate from the ΛCDM, but for $\nu \to 0$ they retrieve their standard forms (in particular, ρ_Λ becomes constant). Recalling that $|\nu| \ll 1$, the model is almost indistinguishable from the concordance ΛCDM, except for its mild vacuum dynamical behavior which leads to an effective

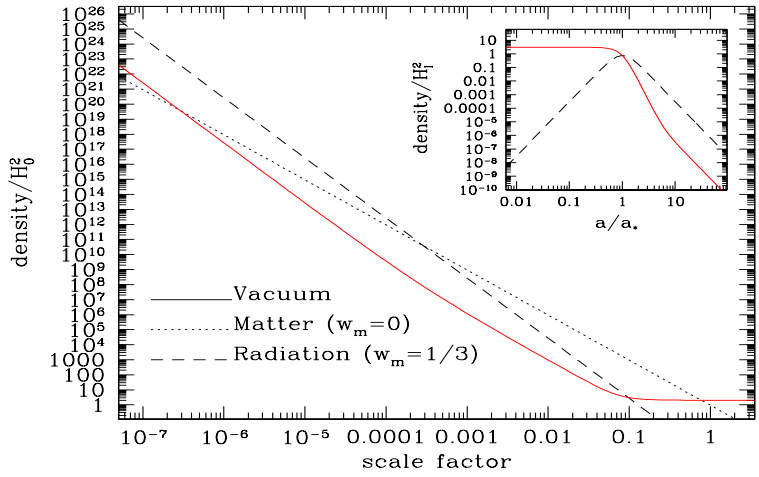

FIG. 1: **Outer Plot:** The evolution of the radiation, non-relativistic matter and vacuum energy densities, for the unified vacuum model (2) with $n = 2$ in units of H_0^2. Note that similar behavior is found also $\forall n$. The curves shown are: radiation (dashed line), non-relativistic matter (dotted line) and vacuum (solid line, in red). To produce the lines we used $\nu = 10^{-3}$, $\Omega_m^0 = 0.3175$, $\Omega_R^0 = (1+0.227N_v)\,\Omega_\gamma^0$, $\Omega_\Lambda^0 = 1-\Omega_m^0-\Omega_R^0$, $(N_v, \Omega_\gamma^0, h) \simeq (3.04, 2.47\times 10^{-5}h^{-2}, 0.6711)$ (cf. Ade *et al.* [15], *Planck results*), and set $\alpha = 1$ and $D = 1/a_*^{3(1-\nu)(1+\omega)}$. **Inner Plot:** the primeval vacuum epoch (inflationary period) into the FLRW radiation epoch. Same notation for curves as before, although the densities are now normalized with respect to H_I^2 and the scale factor with respect to a_*. For convenience we used $8\pi G = 1$ units in the plots.

equation of state that can mimic quintessence or phantom energy [16]. This also means that structure formation after recombination evolves like in the ΛCDM model. At very late times, H becomes constant again: $H \approx H_0\sqrt{(\Omega_\Lambda - \nu)/(1-\nu)}$, hence opening up a new pure de Sitter phase. As an example, in Fig. 1 we display the case $n = 2$, with the mentioned details for the early (inner plot) and intermediate/late (outer plot) stages of the cosmic evolution. For $z \leq 10$ (or $a \geq 0.1$) the vacuum energy density appears virtually frozen to its nominal value, $\rho_\Lambda \approx \rho_\Lambda^0$, close to the matter density.

The upshot is a unified vacuum picture, spanning the entire history of the universe and deviating at present only very mildly from the observed ΛCDM behavior. For any power $n > 1$ in (2), the value of Λ at the early de Sitter phase is $\Lambda_I \sim H_I^2$ while at present $\Lambda_0 \sim H_0^2$. For H_I near the Planck scale, the correct ratio $\Lambda_I/\Lambda_0 \sim 10^{122}$ ensues.

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